International Civil Aviation Organization

The Fourteenth Meeting of the Regional Airspace Safety Monitoring Advisory Group (RASMAG/14)

Bangkok, Thailand, 21 − 25 February 2011

Agenda Item 3: Reports from Asia/Pacific RMAs and EMAs

AIR SPACE ANALYSIS OF BAY OF BENGAL ARABIAN SEA REGION AND SAFETY ASSESSMENT OF RNP 10 ATS ROUTES L510, N571, P628 & P762

(Presented by India)

SUMMARY

The purpose of this working paper is to present the meeting with a summary of the Air Space Analysis of Bay of Bengal Arabian Sea region and safety assessment for the RNP 10 ATS routes L510, N571, P628 and P762 for the introduction of 50 NM (RHS) longitudinal separation.

The qualitative Risk Assessment for the longitudinal collision risk has been carried out using internationally recognized Reich Collision Risk Model. The safety assessment was conducted taking into account the Traffic Sample Data for the month of December 2010, radar surveillance data from all concerned ATC centers and GNE data collected from July to December 2010.

The safety assessment and Air Space Analysis have been carried out by a team of ATM and mathematical experts from Indian Statistical Institute and DGCA the regulator. India (BOBASMA) and Singapore (SEASMA) have been sharing the data and experience with the Objective

a. to confirm that the regionally established target level of safety (TLS) for the airspace has been met prior to the introduction of RHS 50NM and
b. to place on records the credentials of India EMA (BOBASMA) and to demonstrate its competence to receive the formal approval from RASMAG so as to continue the task of discharging the responsibilities as a competent Airspace safety Monitoring Organization.

1. INTRODUCTION

1.1 BBACG/20 meeting (January 2009) recognized the necessity for a formal monitoring program, on a sub-regional basis for lateral and longitudinal navigation errors in the Bay of Bengal to support implementation of Reduced Horizontal plane separation and that limitations in the availability of Safety Monitoring services would hinder implementation. In the first meeting of the Bay of Bengal Reduced Horizontal Separation Task Force (BOB-RHS/TF/1) held during 2^{nd} to 6^{th} November 2009 the terms of reference was amended to include airspace over Arabian Sea. India accepted the responsibility of establishing an En-route Monitoring Agency (EMA).

2. DISCUSSION

2.1. During the third and fourth meetings of the Bay of Bengal Reduced Horizontal Separation Task Force (BOB-RHS/TF/3 & BOB-RHS/TF/4) it was decided to implement 50NM longitudinal separation along four RNAV 10 routes of L510, N571, P628 & P762 in the first phase accordingly in the BOB-RHS/TF/5the members states have given the 'Go' decision. It was further agreed that any introduction of reduced separation minima would be subject to the provisions of Annex 11 safety management system requirements and undergo a safety assessment based on collision risk modeling to confirm that the regionally established target level of safety (TLS) for the airspace has been met.

2.2. This is to highlight the work accomplished by India in preparation for the introduction of 50NM longitudinal separation on RNP 10 routes in the Bay of Bengal, Arabian Sea Region and Indian Ocean. The document attached herewith includes the Airspace analysis as **Appendix-A** and Collision risk assessment as **Appendix-B** in respect of Bay of Bengal Arabian Sea region.

2.3. The quantitative risk assessment for the lateral collision risk has been carried out using internationally recognized Reich Collision Risk Model. The CRM uses Traffic Sample Data (TSD) of December 2010, radar surveillance data from Chennai, Kolkata, Mumbai, Yangon and Kuala Lumpur Air Traffic Control and the Gross Navigational Error data collected from July 2010 to December 2010.

2.4. The safety assessment and Air Space Analysis have been carried out by a team of ATM and mathematical experts from Indian Statistical Institute and DGCA the regulator. India (BOBASMA) and Singapore (SEASMA) have been sharing the data and experience towards achieving the goal.

2.5. The lateral collision risk is estimated to be 6.01881×10^{-10} & the longitudinal collision risk 3.71804×10^{-10}, both of which are well below the TLS of 5×10^{-9}. Thus it can be concluded that the Safety Assessment supports the continued use of 50NM RNP10 lateral separation and also the implementation of RNP10 50NM longitudinal separation on L510, N571, P628 and P762.

3. ACTION BY THE MEETING

3.1 The meeting is invited to:

a. Note the target level of safety (TLS) arrived at through the safety assessment for Bay of Bengal Arabian Sea Region which is well below the TLS prescribed for the airspace under consideration for the introduction of RHS 50 NM.

b. Take note of the credentials presented by India EMA (BOBASMA) to demonstrate its competence to receive formal approval from RASMAG for continuing the task of discharging the responsibilities of EMA and

c. Endorse India EMA (BOBASMA) as a competent Airspace safety Monitoring Agency for Bay of Bengal, Arabian Sea and Indian Ocean.

Intentionally Left Blank

APPENDIX-A

AIRSPACE ANALYSIS

1. INTRODUCTION

The present route network forms part of the EMARSSH project introduced in November 2002 and is mostly composed of nearly parallel North West – south east RNP10 routes catering to the traffic flow between Middle East and Southeast Asia. Another set of routes originate from the busy traffic hub of South East Asia and proceed northwards through Afghanistan to Europe and beyond. The existing non RNAV routes will be phased out shortly. There was no large lateral deviation of flights due navigational error since the implementation of this route structure. The safety assessment carried out by Singapore (SEASMA) with TSD of December 2009 and the results presented in their Working paper in BOB-RHS/TF/5 meeting by SEASMA strongly supports the continued use of safe operation of 50 NM lateral separation over Bay of Bengal and Arabian Sea. There are large weather deviations during the monsoon season over both Bay of Bengal and Arabian Sea.

2. AIRSPACE DESCRIPTION

The airspace analyzed is the Bay of Bengal Arabian sea region where RNP 10 and RVSM are implemented. The four Routes of N571, P628, P762 & L510 are analyzed. P628 is westbound unidirectional & L510 is eastbound unidirectional except during BOBCAT period and N571 is a bidirectional route crossing the entire breadth of the Bay of Bengal Arabian Sea region. P762 is a bidirectional crossing route from North east to South west and it is originating from South East Asia proceeding towards African countries via Colombo. This study is mainly focused on the introduction of Reduced Longitudinal separation of 50NM. Figure 1 shows the existing route network.

The Bay of Bengal Arabian Sea region spreads over twelve Flight Information Regions i.e., Chennai, Mumbai, Kolkata, Bangladesh, Yangon, Bangkok, Jakarta, Kuala Lumpur, Colombo, Male, Seychelles & Muscat

The four routes on which 50 NM reduced longitudinal separation is to be introduced in the first phase are N571, P628, and P762 & L510. Routes N571 & P762 are bidirectional and routes P628 & L510 are unidirectional. P628 is westbound and L510 eastbound unidirectional except when BOB-CAT level allocation takes place. All four routes follow the semicircular system of flight level allocation.

Route N877 is a converging/diverging route with N571 over common way point LAGOG. P761 is a crossing route with N571 over common way point IDASO. P762 is a crossing route with N571 over common way point BIKEN. Presently FL290 and FL320 have been allocated as No PDC level. P762 & P628 cross over common way point PPB.

The oceanic airspace of Bay of Bengal and Arabian Sea is Class E airspace and is subject to procedural control with pilot reporting waypoint either by voice or through data link. At present a longitudinal separation of 10 minutes based on MNT or 80NM is prescribed for routes N571 & P628. For routes L510 & P762 the only separation standard is 10minutes based on MNT. A Longitudinal separation of 15 minutes is prescribed for crossing routes.

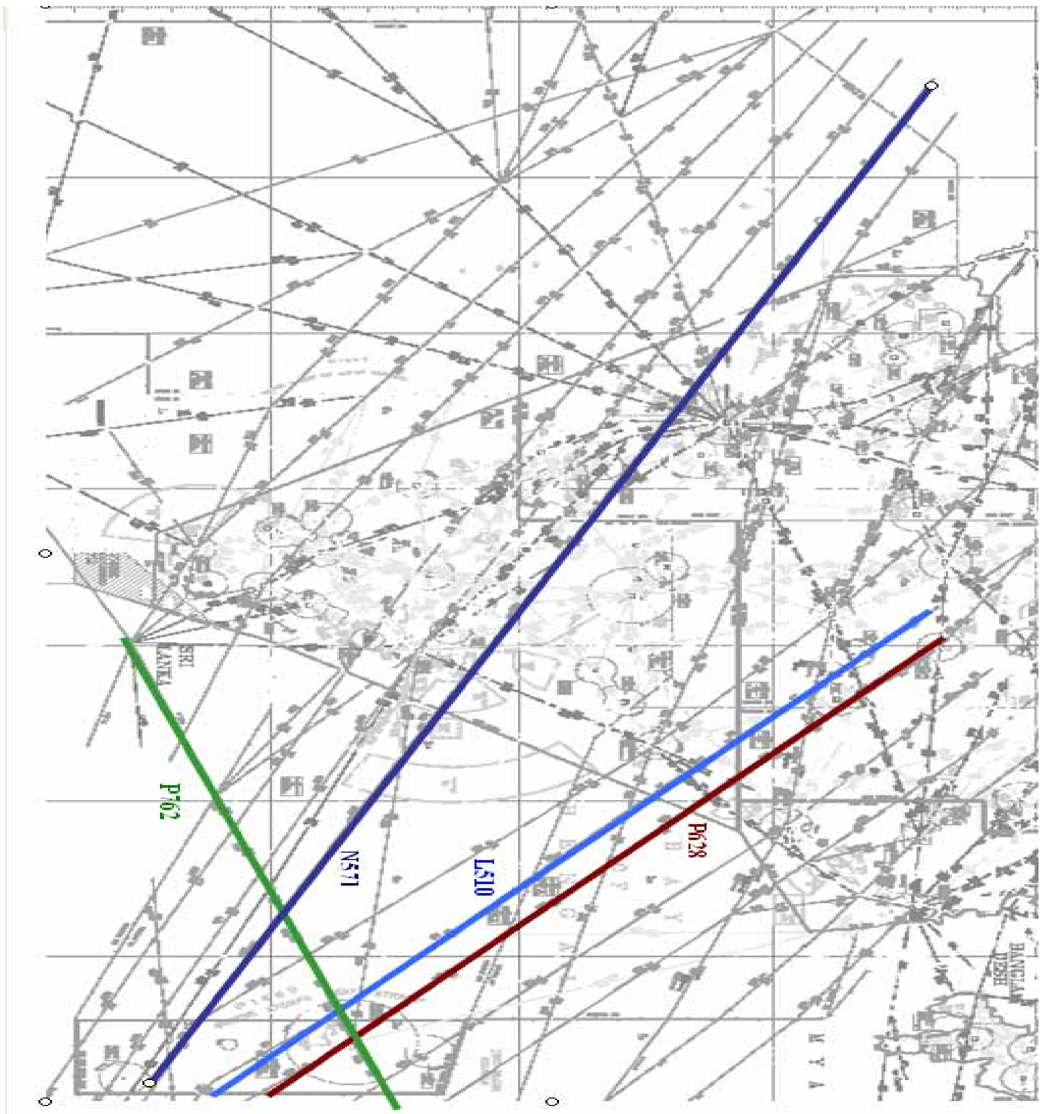

Figure 1

3. COMMUNICATION AND SURVEILLANCE.

The airspace being in the equatorial region, HF communications has inherent operational limitations due to ionospheric effects. CPDLC has proved to be an effective DCPC tool.

RCAG VHF is provided as primary backup frequency for CPDLC in Chennai, Mumbai & Kolkata FIRs and has been found to work satisfactorily. Figure2. Depicts the VHF-RCAG coverage in Indian FIRs. The VHF coverage of Yangon FIR extends up to LALIT on P762 and that of Kuala Lumpur is up to 30 NM short of IGREX, EMRAN & IGOGU. The VHF coverage of Colombo FIR is up to DUGOS on P762.

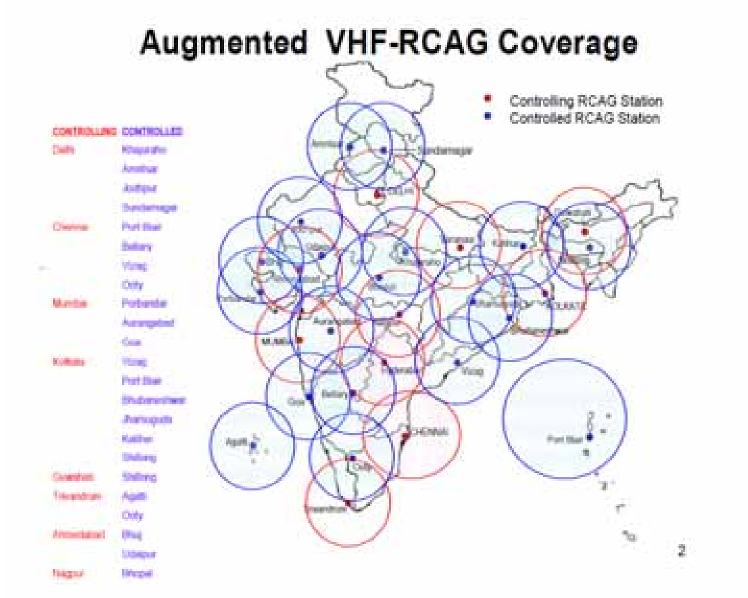

Figure 2. VHF – RCAG Coverage in Indian FIRs.

Radar surveillance is available over the continental airspace. The MSSR radars at Chennai, Mumbai & Kolkata extend up to approximately 200NM into oceanic airspace. These radars provide an opportunity to monitor the lateral deviations of aircraft and surveillance data collected from these radars were used for this study. The VHF and Radar coverage diagram are available in the states Aeronautical Information Publications (AIP).

ADS/CPDLC

Over oceanic airspace surveillance is via ADS/CPDLC. The number of airlines using ADS/CPDLC services within Indian FIRs is around 29. The percentage of aircraft using ADS/CPDLC in Mumbai FIR is 48% and in Chennai FIR is 51% and in Kolkata FIR is 60%. Malaysia has undertaken software upgrades of its ADS/CPDLC system which serve seven oceanic routes P628, L510, L645, L627, N571, B466 & P574 since May 2010 and commenced 24-hour operational trials from 11 October 2010. Maldives had implemented ADS-C/CPDLC services since 2009 and the equipment problem faced was expected to be fixed by March 2010. Myanmar has the ability to provide ADS/CPDLC services along P762. Myanmar has also started to integrate a new ADSC/CPDLC system into their ACC displays at the Yangon ACC. This will allow all of the Yangon FIR to operate in a data-link environment when necessary. Srilanka has taken steps to modernize the existing ATM systems with a fully integrated system (Radar, ADS-C/CPDLC, ADS-B etc).

The data link services are provided on 12 international routes over the Bay of Bengal within Chennai FIR i.e., N877, L510, P628, L759, N571, N563, P762, P574, L896, N564, P761 & L645.

Data link services are provided on 15 international routes over Arabian Sea and Indian Ocean i.e., routes M638, P518, L301, N571, P574, N563, M300, P570, R456, G465, A451, A474, A214, B459, G450 and G424

An analysis of aircraft equipage for RNAV routes N571 P628, P574, P762, P628 N563, L510 and N877 between 1st and 30th August, 2010 was done and the result is shown in Table 1 and Figure 3. The aircraft ADS equipage does not vary much from one FIR to another though it has been found that aircraft logging- in, in one FIR does not log on when in another FIR.

Route	Total no of acft	No of ADS Equipped Acft	No of acft logged on ADS	% of ADS equipped acft
N571	2790	1536	1514	55 %
P574	2025	799	330	40 %
P762	613	395	326	64 %
P628	474	418	358	88 %
N563	454	116	34	26 %
L510	366	324	248	89 %
N877	109	62	52	43%

Table 1. Aircraft ADS equipage.

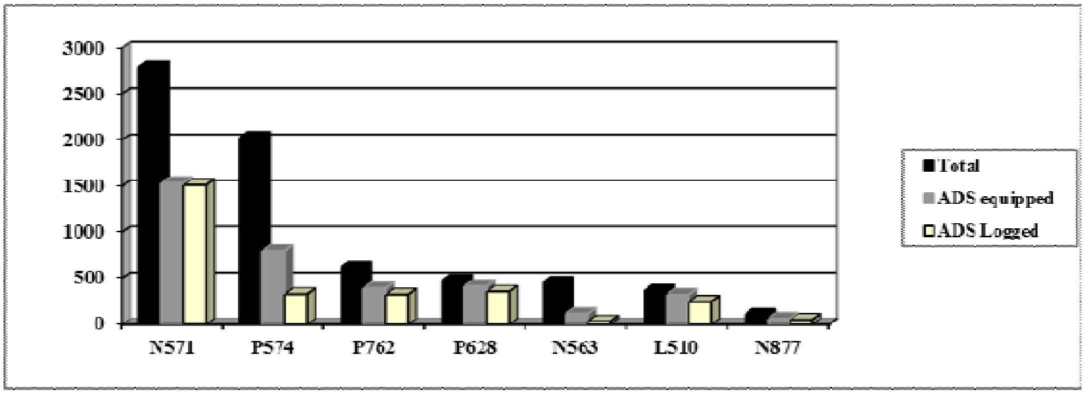

Figure3. Aircraft ADS equipage

TYPE	COUNT	PERCENTAGE	LENGTH	WINGSPAN	HEIGHT
A306	81	1.60	54.1	44.84	16.54
A319	137	2.70	33.84	34.1	11.76
A320	459	9.05	37.57	34.1	11.76
A321	154	3.04	37.57	34.1	11.76
A332	683	13.47	58.8	60.3	17.4
A333	488	9.62	63.6	60.3	16.85
A343	136	2.68	63.6	60.3	16.85
A345	30	0.59	67.9	63.45	17.1
A388	122	2.41	73	79.8	24.1
B722	13	0.26	46.69	32.91	10.36
B737	12	0.24	31.2	35.8	12.6
B738	400	7.89	39.5	35.8	12.5
B742	42	0.83	70.6	59.6	19.3
B743	25	0.49	70.6	59.6	19.3
B744	565	11.14	70.6	64.4	19.4
B74S	28	0.55	76.3	68.5	19.4
B762	14	0.28	48.5	47.6	15.8
B763	30	0.59	54.9	47.6	15.8
B772	751	14.81	63.7	60.9	18.5
B773	95	1.87	73.9	60.9	18.5
B77L	34	0.67	63.7	64.8	18.8
B77W	620	12.22	73.9	64.8	18.7
CL60	10	0.20	20.85	19.6	6.3
F900	13	0.26	20.2	19.3	7.6
GLF4	12	0.24	26.9	23.7	7.4
OTHERS	118	2.33	-	-	-

Table2: Aircraft Population and number of flights per type in Dec. 2010

4. **DATA COLLECTION.**

It was agreed in the first three meetings of the Bay Of Bengal Reduced Horizontal Separation Task Force that Traffic Sample Data (TSD) for the month of December 2010 and Gross Navigational Error data which is to be collected from 1st July 2010 are to be provided to the EMA by the states concerned.

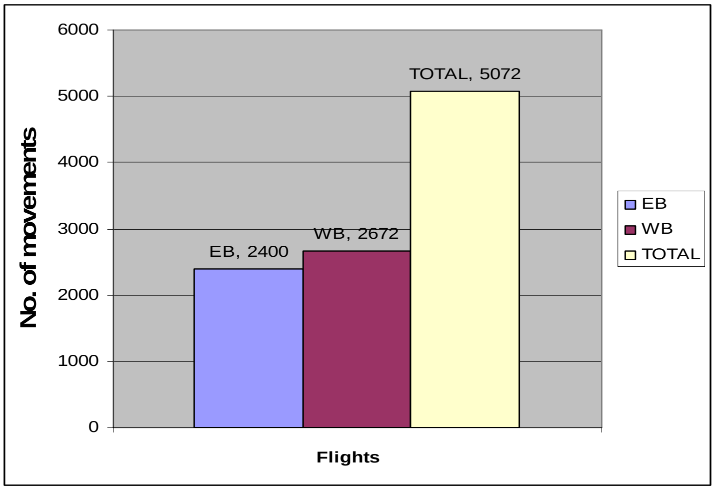

Figure 4. Record of Chennai TSD data collected for the month of December 2010

Route	**Segment**	**FIRs Involved**	**ACCs involved#**
L510	EMRAN & ELBAB	CHENNAI & KOLKATA	CHENNAI, KOLKATA
N571	IDASO & VAMPI	CHENNAI, KUALALUMPUR	CHENNAI
	SUGID & PARAR	MUMBAI, MUSCAT	MUMBAI
P628	LARIK & VATLA	CHENNAI, KOLKATA	CHENNAI, KOLKATA
P762	DUGOS & LULDA	CHENNAI	CHENNAI

Table 3. Monitored fixes in Bay of Bengal Arabian Sea

Month	Report received from							
	India			Colombo	Malaysia	Jakarta	Yangon	Bangkok
	Chennai	Mumbai	Kolkata					
July2010	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Aug2010	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Sep2010	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Oct2010	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Nov2010	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Dec2010	Yes	Yes	Yes	No	Yes	No	Yes	Yes

Table 4. Record of GNE reports for the period July – Dec.2010

Monitoring Month	Cumulative Count of LLEs Reported Over Monitored Fixes Through Monitoring Month	Cumulative Count of LLDs Reported Over Monitored Fixes Through Monitoring Month
July2010	0	0
Aug2010	0	0
Sep2010	0	0
Oct2010	0	0
Nov2010	0	0
Dec2010	0	0

Table5: Monthly count of LLDs and LLEs reported on Bay of Bengal RNAV routes for the period July 2010 to December 2010

5. TRAFFIC ANALYSIS ON L510, N571, P628 & P762.

The latest available Traffic Sample Data of December 2010 from Chennai, Mumbai and Kolkata FIRs have been used to analyze the distribution of flights along the four routes. The analysis based on Type of aircraft, Name of Operator, Distribution of traffic over the days of the week and Flight level usage is displayed in Graphical form.

L510

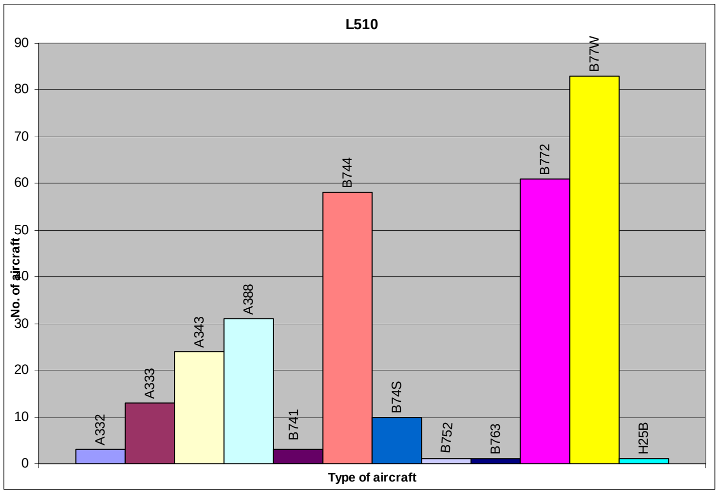

Figure 5 Aircraft Type wise distribution.

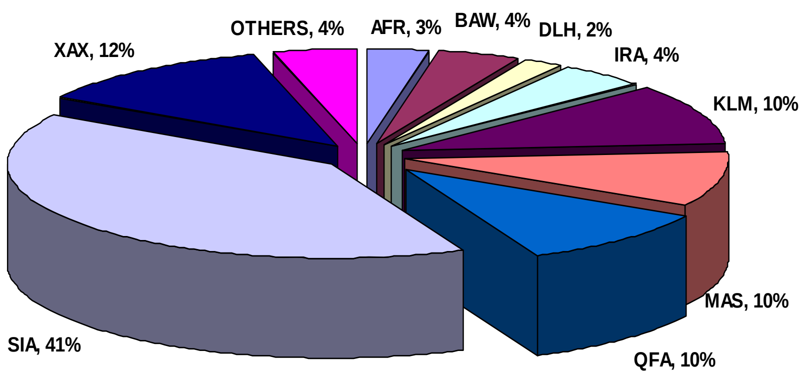

Figure 6 Operator-wise Aircraft Movements

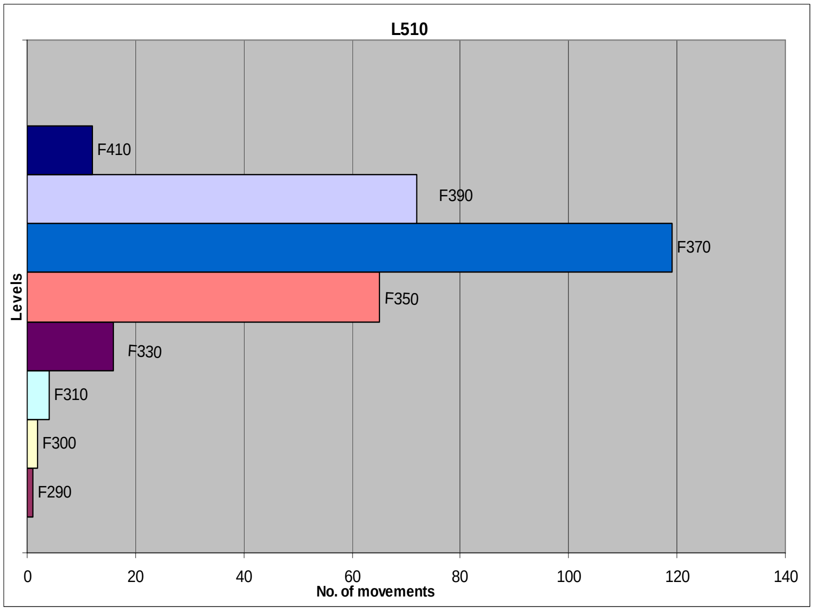

Figure 7 Flight Level usages by aircraft on L510

It can be seen that B77W , B772 & B744 are the three long range aircraft which account for more than 50% of the aircraft using this route. Singapore Airlines operates the maximum number of flights along this route and the next four major operators average about 35 to 40 movements each. The route being unidirectional the most preferred levels are FL370 & FL390 because most of the flights have destinations in Singapore and Malaysia and having already covered more than ¾ th of the flight distance prefer to maintain higher Flight Levels.

<u>N571</u>

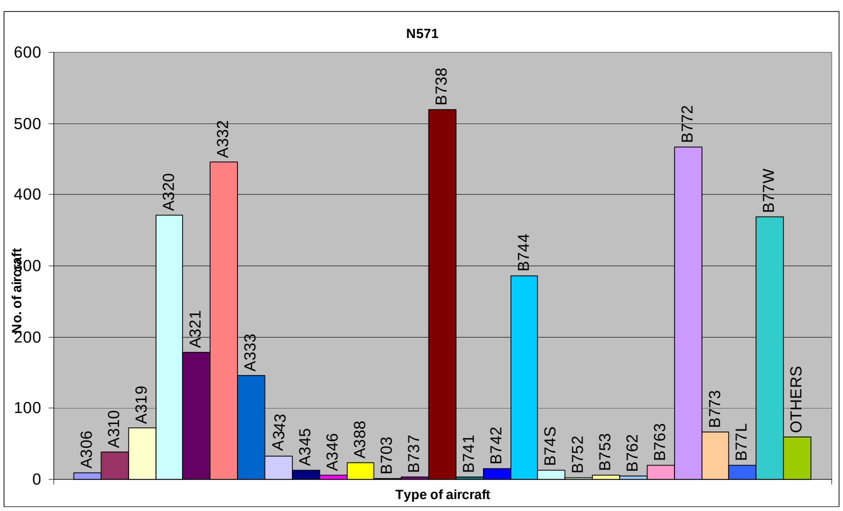

Figure 8 Aircraft Type wise distribution

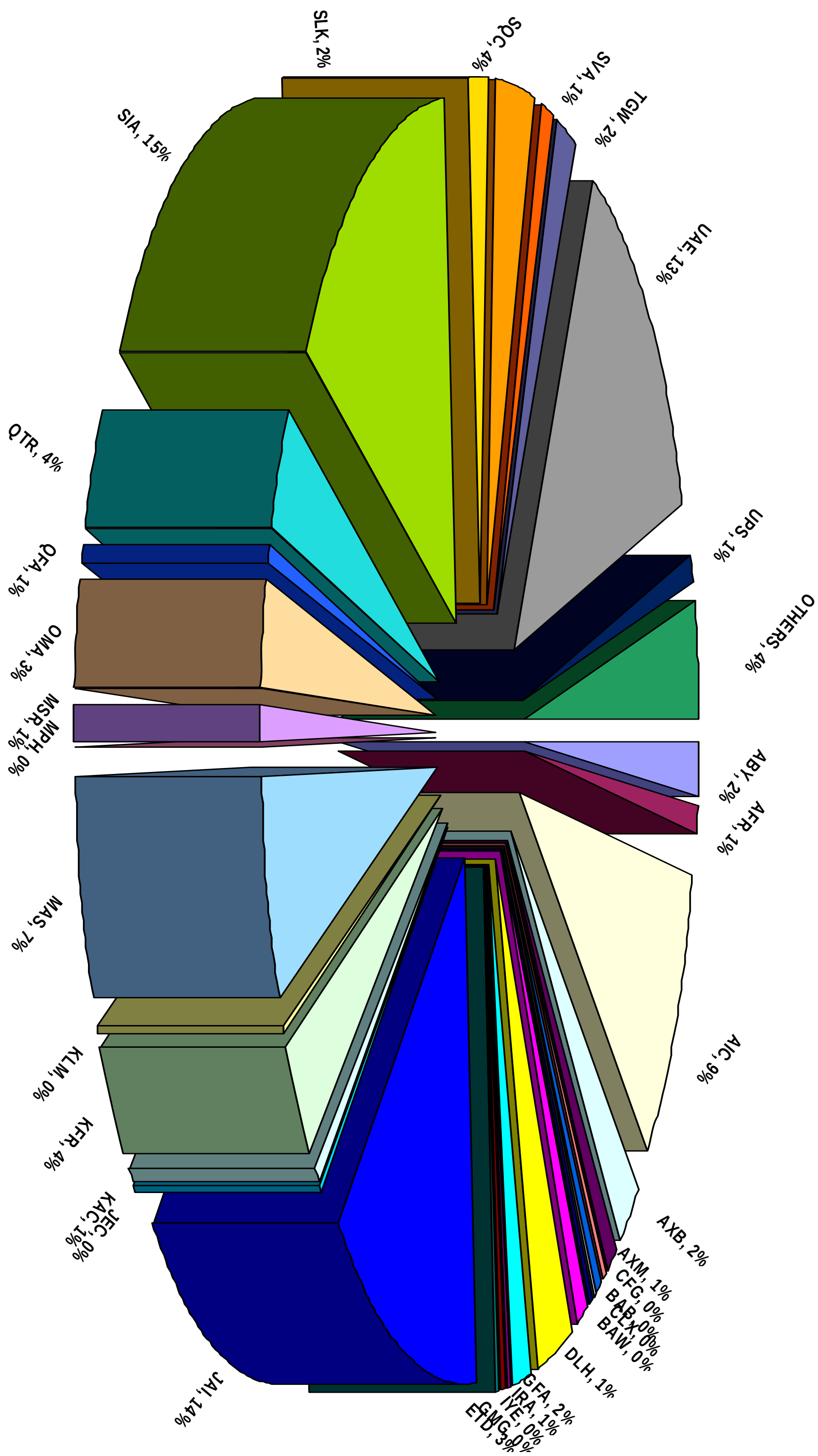

Figure 9 Operator-wise Aircraft Movements

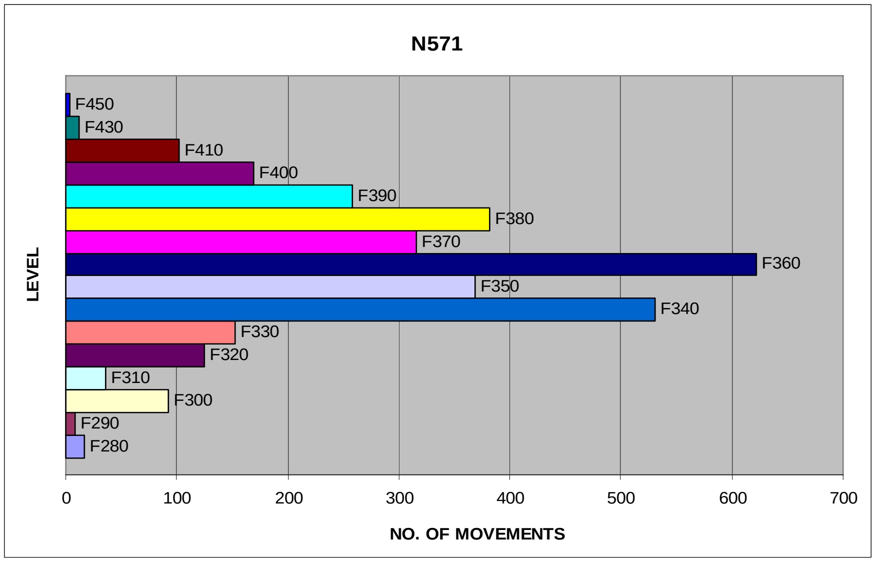

Figure 10 Flight Level usages by aircraft on N571

The maximum usage of this route is by B738. As Singapore Airlines and Emirates who along with Jet airways, are the top three airlines to operate on this route, it can be safely concluded that the maximum number of B738 is due to Jet Airways operating from India to destinations in the middle East and South East Asia and back. Since the Bay of Bengal Arabian Sea region lies midway between the two traffic hubs of Middle East and South East Asia the aircraft tend to maintain the most optimum levels of FL340 to FL380. During severe bad weather conditions in the monsoon season aircraft tend to deviate up to 40 – 50 NM resulting in controllers having to resort to frequent level changes to separate the traffic from those on the parallel route P574 in the Bay of Bengal and from traffic on routes P574 & L301 in the Arabian sea region.

P628

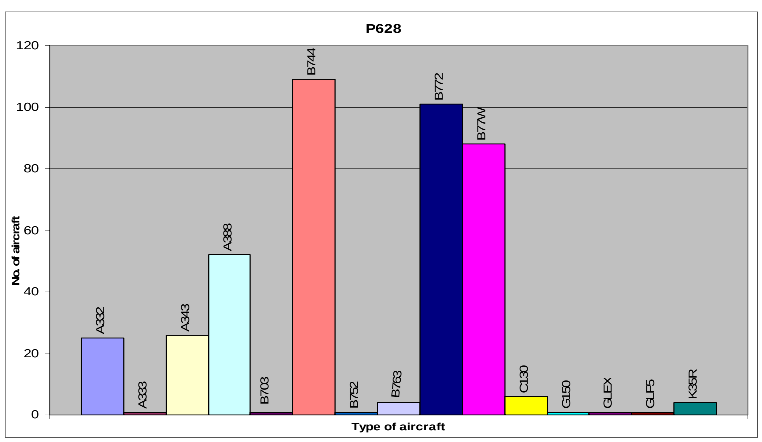

Figure 11 Aircraft Type wise distribution

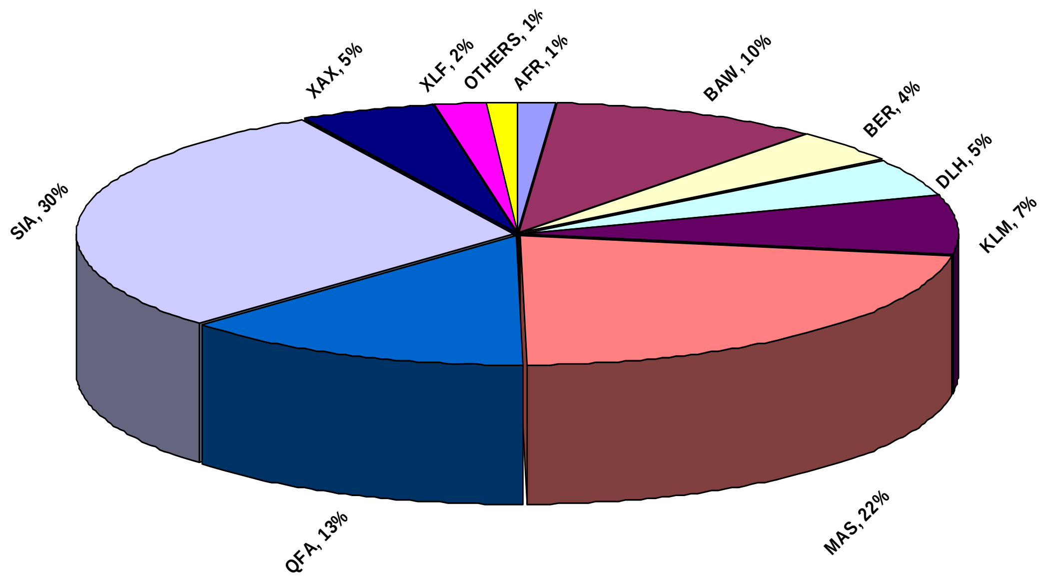

Figure 12 Operator-wise Aircraft Movements

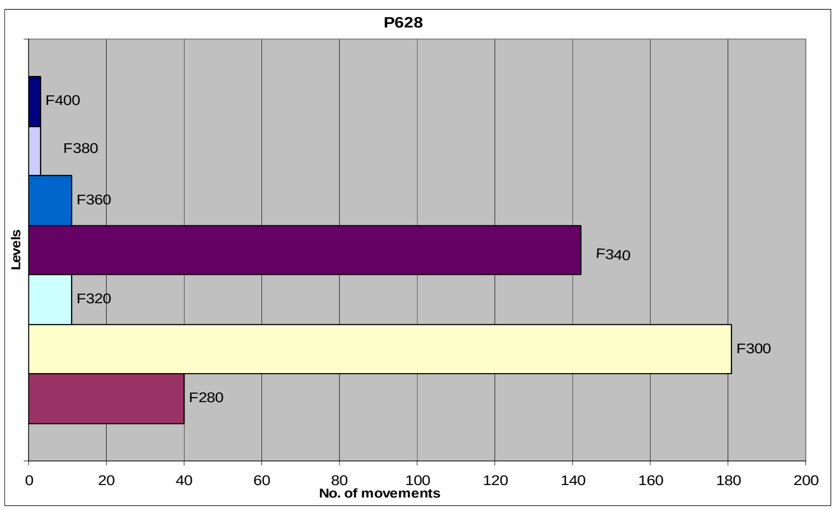

Figure 13 Flight Level usages by aircraft on P628.

It is seen that most of the flights operating on this route originate from either Malaysia or Singapore and mostly fly at FL 340 & below, FL300 being the most favored level. This may be because they are in the initial stage of their flight and may be too heavy to climb to much higher levels. The major aircraft types are B744, B772 & B77W.

P762

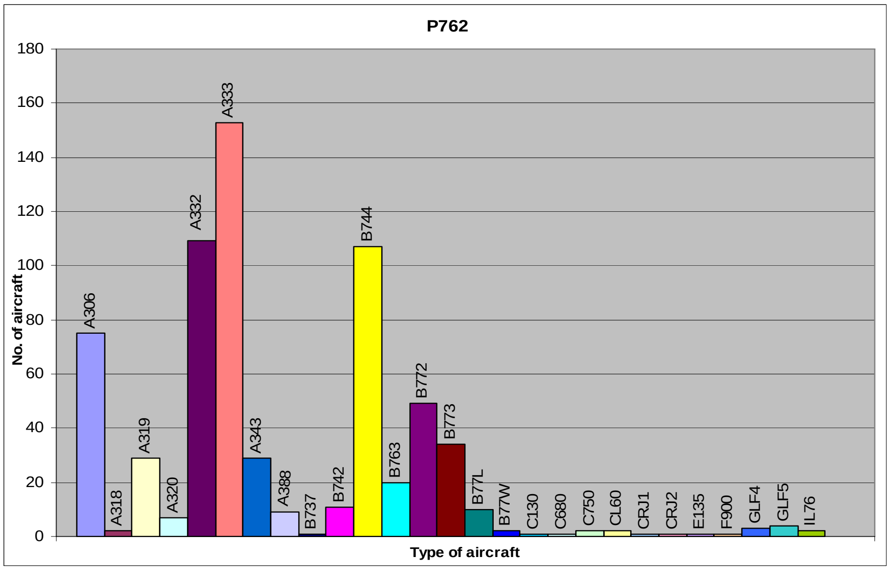

Figure 14 Aircraft Type wise distributions

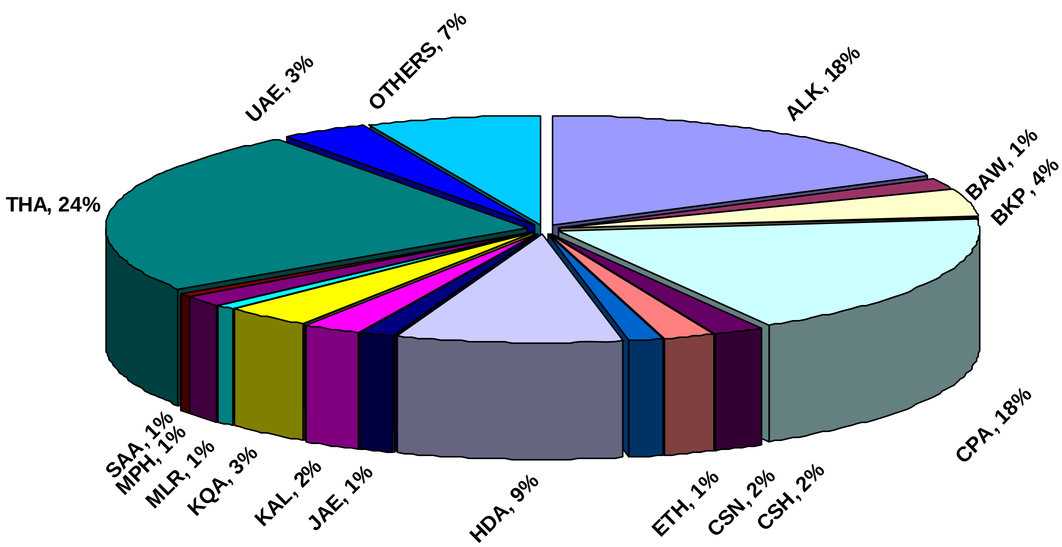

Figure 15 Operator-wise Aircraft Movements

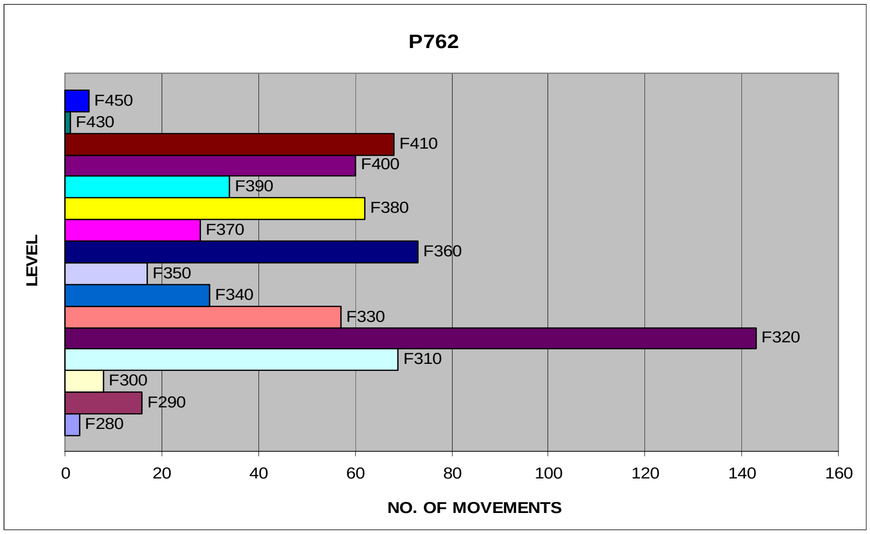

Figure 16 Flight Level usages by aircraft on P762.

P762 caters to the traffic between Bangkok and South Africa. The route cuts across major East – West routes in the Bay of Bengal region. FL320 is the pre-coordinated level between Yangon and Chennai and is also the most preferred westbound level. Thai airways and Cathay Pacific are the major airlines using this route.

Weekly Traffic flow

The weekly traffic flow on the four routes L510, N571, P628 & P762 were analyzed to find the peak traffic days on the different routes.

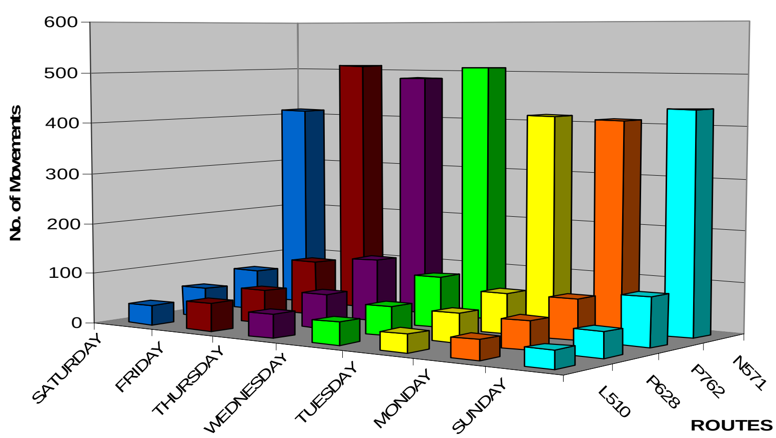

Figure 17 Weekly movements on four routes.

Traffic Density.

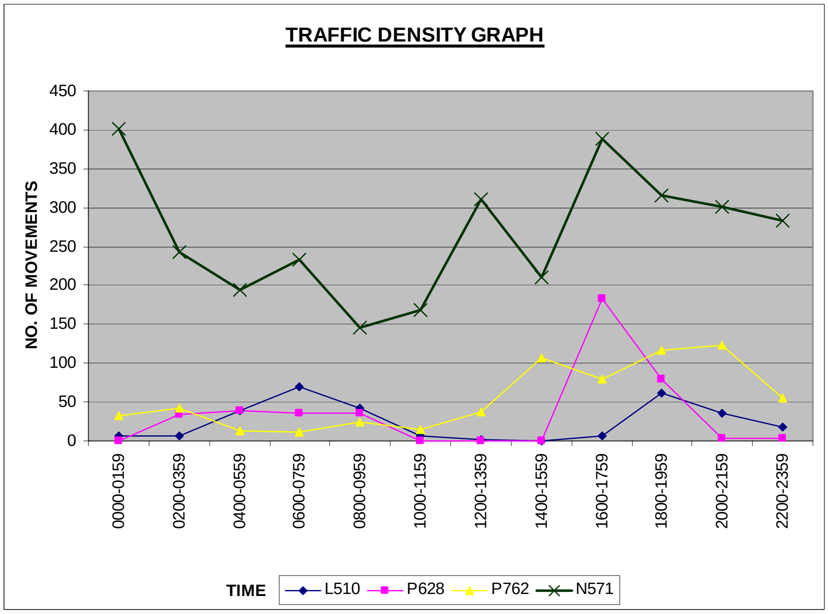

Figure 18 Traffic density pattern on the four routes.

6. WEATHER DEVIATION AND CONTINGENCY PROCEDURES

Between June to September the Arabian Sea region experiences south – West Monsoon climatic conditions and the Bay of Bengal region experiences north – East monsoon conditions between September to November. During this period aircraft are not able to maintain assigned flight level and track and need to deviate either left or right of track. Such deviations at times can be up to 70 to 80 NM.

A study was undertaken for the Bay of Bengal region using 20 days data from 28th October 2009 to 15th November 2009. Figure2 shows that the extent of deviation varied from 10NM to 60NM during this period. Number of aircraft deviations by more than 30NM from the intended track comes around 34, within a span of 15 days.

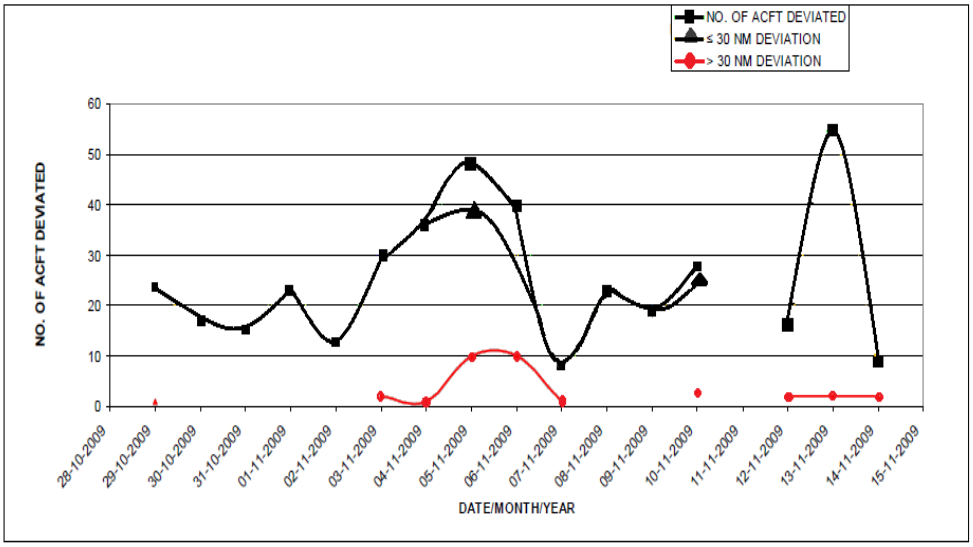

Figure 19. Weather Deviation over Bay of Bengal.

AIP India prescribes Weather deviation procedures for such contingencies. Aircraft are essentially required to either descend/climb by 300FT, depending on the direction of flight when unable to establish contact with ATC and the deviation is beyond 10NM. If deviation is to be less than 10NM aircraft shall maintain assigned level.

AIP India prescribes Weather deviation procedures for such contingencies. Aircraft are essentially required to either descend/climb by 300FT, depending on the direction of flight when unable to establish contact with ATC and the deviation is beyond 10NM. If deviation is to be less than 10NM aircraft shall maintain assigned level.

7. BENEFITS OF REDUCED HORIZONTAL SEPARATION.

The introduction of Reduced Horizontal Separation is bound to enhance Safety, airspace capacity, increase fuel efficiency and reduce aircraft emissions.

Safety

Implementation of RHS will enhance safety in two ways.

(i) By increased surveillance using ADS/CPDLC as more and more aircraft are encouraged to have FANS1/A equipage.

(ii) Increased DCPC capability through enhanced VHF coverage and CPDLC.

8. AIRSPACE CAPACITY.

Table 6. shows the extent of capacity increase that can be achieved.

YEAR	ROUTE(S) & DISTANCE (NM)	SPACING BTN ACFT LAT/LONG (NM)	AIRSPACE / NO. OF VERTICAL LAYERS (FLIGHT LEVELS)	NO. OF AIRCRAFT
BEFORE 2002	SINGLE NON-RNP 2050	>200/120	CVSM(2000FT) / 9	153
2002	3 PARALLEL – RNP-10 2000 X 3	50/80	CVSM(2000FT) / 9	675
2003	3 PARALLEL – RNP-10 2000 X 3	50/80	RVSM(1000 FT) / 15	1125
2011	3 PARALLEL RNP-10 2000 X 3	50/50	RVSM(FANS) / 15	1800
2012*	4 PARALLEL RNP-4 2000X 4	30/30	RVSM(FANS) / 15	4000

Table 6.

9. FUEL EFFICIENCY

Fuel efficiency is one of the most import criteria in flight operations. It is estimated that for every 2000 ft difference from optimum flight level, the aircraft burns out approximately an additional 2% of fuel per hour. Hence for example, a heavy aircraft like Boeing 747, which burns 3.79 litres of fuel per second, flying at 2000 feet above or below its optimum level, from South East Asian countries to Middle East or to cross Middle East towards Europe, in its EET of 6 hr. 45 minutes, burns out an extra 1847 litres (2 %) during the flight.

10. ENVIRONMENTAL BENEFITS

Carbon dioxide (CO2), nitrogen oxides (NOX), sulfur oxides (SOX), and particulates (soot), are examples of aircraft emissions which may alter atmospheric processes. A scientific assessment published by the Intergovernmental Panel on Climate Change (IPCC) attributes 3.5% of the total harmful effects on environment resulting from human activities to aviation and suggests that the impact of aircraft emissions at altitude is potentially twice as severe with respect to climate change when compared to ground level emissions.

Amount of toxic gases emitted by burning of fuel, as estimated by US EPA (United States Environmental Protection Agency) is given below :

CO_2 (kg) = 2.56 x amount of fuel burned (lit);

SOx (kg) = 0.00065 x amount of fuel burned (lit)

NOx (kg)=0.0005 x amout of fuel burned (lit)

Hence wastage of 1847 litres of fuel, as per the existing traffic, adds 2.5 million kg of CO_2, and 634 Kg of SOx and a similar amount of NOx, to the atmosphere per month.

11. IMPACT ON ATS ROUTES LINKING INDIA WITH THE MIDDLE EAST VIA THE ARABIAN SEA

The routes transiting Bay of Bengal Arabian Sea regions also pass through the radar coverage area of Muscat FIR and beyond. When releasing eastbound traffic into Mumbai FIR Muscat FIR has to expand the separation between aircraft at the same level from 5 – 10 NM to 80NM. With the introduction of RHS Muscat FIR would have that much less workload.

12. KABUL FIR

The primary mission of the Kabul ACC and ATC services in Afghanistan was to support the coalition forces in Afghanistan and this affects handling of over-flight traffic to and from the south Asian sub-region. Kabul FIR still remains a procedural control facility and the standard longitudinal separation used is 80NM. The ATFM BOBCAT arrangement is mainly to support Kabul FIR in handling the traffic congestion on these routes. The introduction of 50NM longitudinal separation on routes leading from South East Asia to Europe and beyond may lead to even more traffic build up at the entry point into Kabul FIR. If Kabul also implements 50NM reduced longitudinal separation it would ease the traffic flow through Afghanistan airspace and it would also be possible to decrease the spacing presently used by the BOBCAT system.

13. CROSSING ROUTES

The primary traffic flow in the Bay of Bengal region is between South East Asia and India. The crossing routes serving traffic between Asia and Africa though not as heavily loaded, still affects the allocation of flight levels on the primary routes. These limitations can be overcome by establishing unidirectional routes (All levels available) for the crossing traffic, and aircraft on these routes could be allocated the same levels, which will effectively sustain the capacity on the primary routes.

14. SAFETY OVERSIGHT AND RECOMMENDATION

The lateral collision risk is estimated to be 6.01881×10^{-10} & the longitudinal collision risk 3.71804×10^{-10}, both of which are well below the TLS of 5×10^{-9}. Thus it can be concluded that the Safety Assessment supports the continued use of 50NM RNP10 lateral separation and also the implementation of RNP10 50NM longitudinal separation on L510, N571, P628 and P762.

APPENDIX-B

LATERAL AND LONGITUDINAL COLLISION RISK ASSESSMENT OF BAY OF BENGAL

1. INTRODUCTION

In this article we investigate the collision risk between two aircraft flying over the Bay of Bengal. This safety assessment is undertaken by the Airports Authority of India (AAI). The goal of this study is to confirm that the Target Level of Safety (TLS) which is 5×10^{-9} collisions per flight hour is currently met. This analysis will also help the AAI to establish En-route Monitoring Agency (EMA) which is necessary for further lateral and longitudinal separation reductions. It is important to note that currently the separation standards are as follows

- for lateral separation it is at least 50 NM between all the parallel routes;
- for longitudinal it is at least 10 minutes leading to an average 80 NM between front and behind aircraft.

In this article we carry out the quantitative risk analysis based on two types of data sets collected by the AAI.

• Traffic Sample Data (TSD):

Traffic sample data from Chennai FIR for the month of December 2010 was used. The original sample contained 4986 records. The data contained several anomalies, which we tried to detect and remove. We also removed entries for routes that are not relevant for the current study. Briefly, the following initial filtering criteria were used:

- Records with Exit time less than Entry time were removed.
- Records with missing data on entry/exit points, entry/exit levels, entry/exit times were removed.
- Records with flight level less than F290 were removed.
- Records whose entry/exit routes were inconsistent were removed.
- Only records for routes L510, P628, N877, P574, N571, and P762 were retained.

2741 flights that were retained after filtering were considered for the subsequent statistical analysis and Figure 1 provides a more detailed graphical summary with additional break-up by flight level and direction.

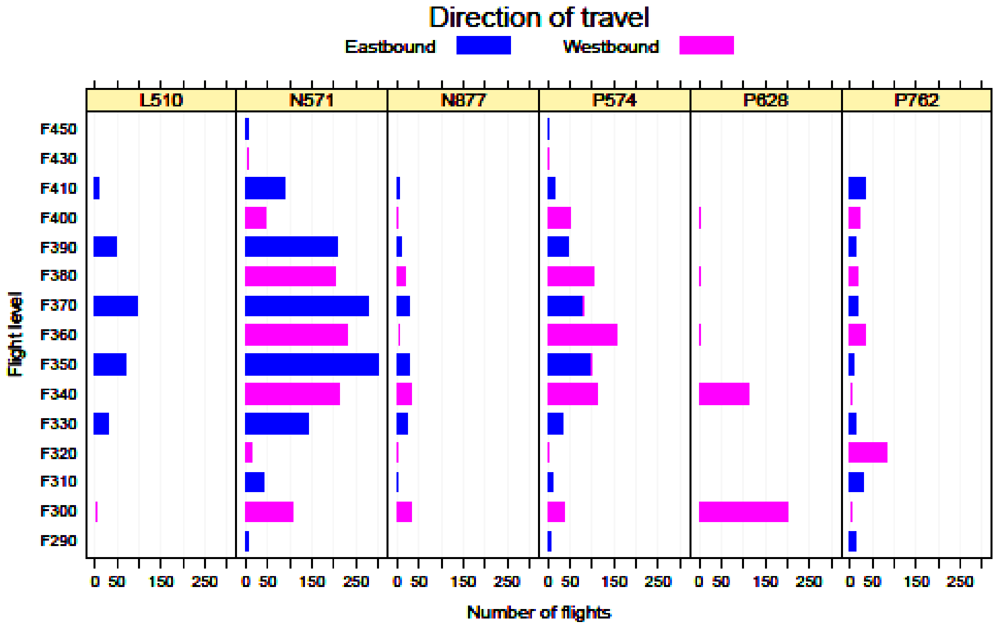

Figure 1: Number of flights per route and flight level entering Chennai FIR, based on December 2010 TSD.

Gross Navigational Error (GNE) Data:

Reports of Gross Navigational Errors were received from India (Chennai, Mumbai, and Kolkata FIRs) and Bangkok for the months of July to December 2010, as summarized in Table1. In Section 2 we discuss the risk assessment for the lateral direction and in Section 3 gives the same for the longitudinal direction.

Year	**Month**	**FIR**	**Flights**	**LLE**	**LLD**
2010	AUGUST	KOLKATA	443	0	0
2010	SEPTEMBER	KOLKATA	423	0	0
2010	OCTOBER	KOLKATA	432	0	0
2010	NOVEMBER	KOLKATA	427	0	0
2010	DECEMBER	KOLKATA	545	0	0
2010	JULY	CHENNAI	2679	0	0
2010	AUGUST	CHENNAI	5173	0	0
2010	SEPTEMBER	CHENNAI	5196	0	0
2010	OCTOBER	CHENNAI	5478	0	0
2010	NOVEMBER	CHENNAI	5258	0	0
2010	DECEMBER	CHENNAI	5432	0	0
2010	JULY	MUMBAI	1838	0	0
2010	AUGUST	MUMBAI	1812	0	0
2010	SEPTEMBER	MUMBAI	1792	0	0
2010	OCTOBER	MUMBAI	1884	0	0
2010	NOVEMBER	MUMBAI	1068	0	0

2010	DECEMBER	MUMBAI	1426	0	0
2010	JULY	BANGKOK	1865	0	0
2010	AUGUST	BANGKOK	2330	0	0
2010	SEPTEMBER	BANGKOK	2297	0	0
2010	OCTOBER	BANGKOK	2234	0	0
2010	NOVEMBER	BANGKOK	2108	0	0
2010	DECEMBER	BANGKOK	2061	0	0
2011	JANUARY	BANGKOK		0	0

Table1: Summary of reports of Gross Navigational Errors.

2 LATERAL COLLISION RISK ASSESSMENT

2.1 Lateral Collision Risk Model

In order to compute the *level of safety* for lateral deviations of operations on the Bay of Bengal we use the *Reich Lateral Collision Risk Model*. It models the lateral collision risk due to the loss of lateral separation between aircraft on adjacent parallel tracks flying at the same flight level. The model is as follows:

$$N_{ay} = P_y(S_y)\,P_z(0)\,\frac{\lambda_x}{S_x}\left\{E_y(\text{same})\left[\frac{|\Delta\bar{V}|}{2\lambda_x}+\frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y}+\frac{|\bar{\dot{z}}|}{2\lambda_z}\right]+E_y(\text{opp})\left[\frac{|2\bar{V}|}{2\lambda_x}+\frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y}+\frac{|\bar{\dot{z}}|}{2\lambda_z}\right]\right\}. \quad (1)$$

We would like to note that same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA and also in European safety assessment which was carried out for EUR/SAM corridor.

The parameters in the equation (1) are defined as follows:

- Nay := Expected number of accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircraft flying on tracks with planned Sy NM lateral separation.
- Sy := Minimum planned lateral separation.
- λx := Average length of an aircraft flying on Bay of Bengal.
- λy := Average wingspan of an aircraft flying on Bay of Bengal.
- λz := Average height of an aircraft flying on Bay of Bengal.
- Py (Sy) := Probability that two aircraft assigned to two parallel routes with Sy NM lateral separation will lose all planned lateral separation.
- Pz (0):= Probability that two aircraft assigned to same flight level are at same geometric height.
- Sx := Length of half the interval in NM used to count proximate aircraft at adjacent routes.
- Ey (same):= Same direction lateral occupancy at same assigned flight level.
- Ey (opp):= Opposite direction lateral occupancy at same assigned flight level.
- $|\Delta \bar{V}|$ = Average relative speed of two aircraft flying on parallel routes in same direction.

- $\left|\overline{V}\right|$ = Average ground speed on an aircraft.
- $\left|\overline{\dot{y}}(S_y)\right|$ = Average relative lateral speed of aircraft pair at loss of planned lateral separation of S_y.
- $\left|\overline{\dot{z}}\right|$ = Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

A collision and consequently two accidents can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation 1 gathers the product of the probabilities of losing separation in each one of the three dimensions.

As it has already been said, $P_z(0)$ is the probability of vertical overlap; $P_y(S_y)$ is the probability of lateral overlap and the combinations of $\frac{\lambda_x}{S_x}E_y$ (same) and $\frac{\lambda_x}{S_x}E_y$ (opp) relate to the probability of longitudinal overlap of aircraft on adjacent parallel tracks and at the same flight level. All the probabilities can be interpreted as proportions of flight time in the airspace during which overlap in the pertinent dimension occurs. As the collision risk is expressed as the expected number of accidents per flight hour, the joint overlap probability must be converted into number of events involving joint overlap in the three dimensions, relating overlap probability with passing frequency. Here we note that passing frequency between two adjacent routes is the average number of events, per flight hour, in which two aircraft are in longitudinal overlap when travelling in the opposite or same direction at the same flight level. This is achieved by means of the expressions within square brackets in Equation 1. Each of the terms within square brackets represents the reciprocal of the average duration of an overlap in one of the dimensions. For example, $\frac{\left|\Delta\overline{V}\right|}{2\lambda_x}$ is the reciprocal of the average duration of an overlap in the longitudinal direction for same direction traffic. In the case of longitudinal direction too, but for opposite direction, the average relative speed is $2\overline{V}$ and the average overlap time is $\frac{\left|\Delta\overline{V}\right|}{2\lambda_x}$.

The model is based on the following hypothesis:

- All routes are parallel. (In the Bay of Bengal sea area there are cross route, such as, P762. We are excluding these routes from the current study)
- All collisions normally occur between aircraft on adjacent routes, although, if the Probability of overlap is significantly large, they may also occur on non-adjacent routes.
- The entry times into the track system are statistically independent.
- The lateral deviations of aircraft on adjacent tracks are statistically independent.
- The vertical, longitudinal and lateral deviations of an aircraft are statistically independent.
- The aircraft are replaced by rectangular boxes.

• There is no corrective action by pilots or ATC when two aircraft are about to collide.

The model also assumes that the nature of the events making up the lateral collision risk is completely random. This implies that any location within the system can be used to collect a representative data sample on the performance of the system.

2.2 Estimated Values of the Parameters and Estimated Lateral Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (1) Which are obtained from our analysis.

Parameter	**Estimated Values**	**Source of the Estimate**
Sy	50 NM	Current minimum lateral separation
λx	0.0326051NM	Estimated from the TSD of December, 2010 (see Section 2.3).
λy	0.02983705 NM	Estimated from the TSD of December 2010 (see Section 2.3).
λz	0.009069301 NM	Estimated from the TSD of December 2010 (see Section 2.3).
P_y (50)	4.31577×10^{-8}	Estimated using a mixture model (see Section2.4).
P_z (0)	0.3617939	Estimated using a Double Exponential model (see Section 2.5).
S_x	80 NM	Equivalent to ±10-minutes of longitudinal separation.
E_y(same)	0.04880429	Estimated from the TSD of December 2010 (see Section 2.6).
E_y (opp)	0	No opposite directional lateral occupancy at same assigned flight level.
$\left\|\Delta \overline{V}\right\|$	36 knots	Value obtained from TSD (see Section 2.8).
$\left\|\dot{\bar{y}}(50)\right\|$	75 knots	Conservative value taken from EMA Handbook (see Section 2.9).
$\left\|\dot{\bar{z}}\right\|$	1.5 knots	Conservative value as per EMA Handbook(see Section 2.10).

Finally this leads to the following estimate for the lateral collision risk Nay .

$$N_{ay} = 6.01881 \times 10^{-10}$$

2.3 Estimating Average Aircraft Dimensions

We computed the average aircraft dimensions using the dimensions of each aircraft type weighted by their relative proportions

2.4 Estimating Probability of Lateral Overlap: $P_y(S_y)$

The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y, is denoted by $P_y(S_y)$ and is defined as

$P_y(S_y) := P(|S_y + Y_1 - Y_2| \leq \lambda y)$, (2) where Y_1 and Y_2 are assumed to be the lateral deviations of two aircraft which are nominally separated by S_y. We assume that Y_1 and Y_2 are identically distributed but statistically independent with a distribution F_y.

We model F_y as mixture distribution having a core distribution Gy and a non-core distribution H_y.

- The core distribution Gy, represents errors that derive from standard navigation system deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision.
- The non-core distribution H_y, represents Gross Navigation Errors (GNE), that corresponds to what may be viewed as non-nominal performance.

We assume that a standard navigation system error represented by the core distribution may take large values but the non-core distribution representing gross navigation errors can only take large values. But in most cases it is impossible to determine with certainty if a given observed lateral error arose from the core or from the tail term of the distribution. Therefore, the overall lateral deviation distribution is modeled as:

$$F_y(y) = (1 - \alpha)\, G_y(y) + \alpha H_y(y)\,. \quad (3)$$

The mixing parameter α is the probability of a *gross navigational error.*

The core lateral deviation distribution Gy is modeled by a Double Exponential distribution with a parameter βy > 0 as the rate, that is, if Y1 ~ Gy then

$$P(|Y1| > y) = e^{-\beta_y y}$$

In other words we assume that the core distribution has a density of the form

$$g_y(y) = \frac{\beta_y}{2} e^{-\beta_y y}$$

Finally the non-core distribution H_y is modeled by a *"Separated Double Exponential "* distribution with parameters $\mu_y > 0$, representing the "separation and $\gamma_y > 0$ the rate parameter, that is, if Y2 ~ Hy then

$$P(Y2 > \mu_y + y) = \frac{1}{2} e^{-\gamma_y y} \text{ and}$$

$$P(Y2 < -\mu_y - y) = \frac{1}{2} e^{\gamma_y y}$$

This really means that the non-core distribution H_y gives no mass in $[-\mu_y, \mu_y]$ and outside it decays as a Double Exponential distribution with rate parameter γ_y. The density of this distribution is given by

$$h_y(y) = \begin{cases} \frac{\gamma_y}{2} e^{\gamma_y (y+\mu_y)} & \text{if} \quad x < -\mu_y \\ 0 & \text{if} \quad -\mu_y \le x \le \mu_y \\ \frac{\gamma_y}{2} e^{\gamma_y (y-\mu_y)} & \text{if} \quad x > \mu_y \end{cases} .$$

This modeling is similar but more realistic than what has been used by FAA and also in EUR/SAM. The parameter α is estimated by taking the 95% upper confidence limit from the July December 2010 GNE data as provided by AAI. The formula comes out to be

$$\widehat{\alpha} = 1 - (0.05)^{1/N} = 5.526927 \times 10^{-5} ,$$

where N = 54201 is the number of flights observed and no gross navigational errors were detected. More GNE data with no detected gross navigational error will increase the value of N and hence decrease the value of α which will lead to decrease in the risk.

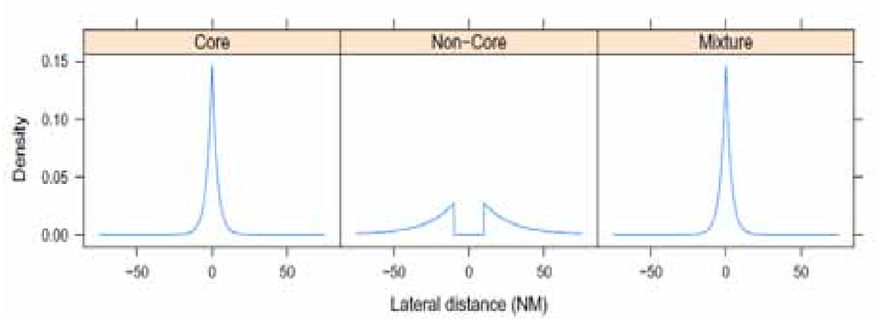

Figure 2: Modeling of lateral deviation.

The parameter β_y is estimated under the RNP10 assumption of ±10 NM deviation with 95% confidence, this leads to the estimate

$$\widehat{\beta_y} = -\frac{\log 0.05}{10} = 0.299573227 .$$

The parameter μ_y is taken to be 10 based on RNP10 consideration and γ_y is then estimated by maximizing the wingspan overlap probability with S_y = 50 NM initial separation. This is a conservative method similar to what has been used by FAA and also in EUR/SAM. The estimated value of γ_y is 0.05489709 leading to the estimated value of P_y (50) as 4.31577×10^{-8}.

2.5 Estimating Probability of Vertical Overlap: P_z (0)

The probability of vertical overlap of aircraft nominally flying at the same flight level on laterally adjacent flight paths is denoted by P_z (0). It is defined by

$$Pz(0) = \mathbf{P}(|Z_1 - Z_2| \le \lambda_z),$$

where Z_1 and Z_2 are the height deviations of two aircraft nominally flying at the same flight levels on laterally adjacent flight paths.

We assume that Z_1 and Z_2 are statistically independent with distribution F_z . Unlike in the computation of P_y (S_y) where we assume the lateral deviations follow a mixture distribution here we only assume that F_z is a Double Exponential distribution with parameter $\beta_z > 0$, that is, with density function

$$f_z(z) = \frac{\beta_z}{2} e^{-\beta_z |z|} .$$

We estimate $\beta_z > 0$ by

$$\widehat{\beta_z} = -\frac{\log 0.05}{0.032915} = 91.014196371 .$$

This is under assumption that a typical aircraft stays within ±200 ft = ±0.032915 NM of its assigned flight level 95% of the time.

2.6 Estimating the Lateral Occupancy Parameters: Ey (same) and Ey (opp)

In equation 1 there are two occupancy terms, one for same direction occupancy Ey (same) and another one for opposite direction occupancy Ey (opp). Same direction occupancy is defined as the average number of aircraft that are, in relation to a typical aircraft

Count By	Routes	Waypoints	Total	Proximate
Entry	(N877, L510)	(ORARA, BIDEX)	316	2
Entry	(N877, P628)	(IGOGU, IGREX)	389	40
Entry	(P574, N571)	(NOPEK, IGOGU)	1188	80
Entry	(P574, N571)	(GIRNA, IDASO)	1254	38
Exit	(N877, P628)	(ORARA, VATLA)	389	20
Exit	(N877, L510)	(IGOGU, EMRAN)	81	0
Exit	(P574, N571)	(NOPEK, IGOGU)	1276	82
Exit	(P574, N571)	(GIRNA, IDASO)	1254	38

Table2: Number of laterally proximate flights per route pair,
based on Chennai FIR December 2010 TSD.

- flying in the same direction as it;
- nominally flying on tracks one lateral separation standard away;
- nominally at the same flight level as it; and
- within a longitudinal segment centered on it.

The length of the longitudinal segment, $2S_x$, is usually considered to be the length equivalent to 20 minutes of flight resulting to a value of 160 NM. It has been verified that the relationship between S_x and the occupancy is quite linear.

A similar set of criteria can be used to define opposite direction occupancy, just replacing "flying in the same direction" by "flying in the opposite direction". Occupancy, in general, relates to the longitudinal overlap probability and can be obtained by the equation

$$E_y = \frac{2T_y}{H},$$

where T_y represents the total proximity time generated in the system and H is the total flight hours generated in the system during the considered period of time. We estimate this quantity by direct estimation from time at waypoint passing using the TSD. For this we compute the number of proximate pairs by comparing the time at which an aircraft on one route passes a waypoint with the time at which another aircraft on a parallel route passes the homologous waypoint. When the difference between passing times is less than certain value, 10 minutes in this case, it is considered that there is a proximate pair in that pair of routes. Occupancy is then calculated using the following expression:

$$E_y = \frac{2n_y}{n},$$

where the numerator ny is the number of proximate pairs and the denominator, n, is the the total number of aircraft. The observed number of proximate pairs and the total number of flights per route pair are summarized in Table 2.

2.7 Estimate of Average Ground Speed

As directly measured speed data were not provided, speeds and relative velocities have been estimated by comparing waypoint report times. To do this, we divide the distance separating the entry and exit waypoints on a route by the time taken to travel the route. The result of this operation is the speed of each aircraft.

2.8 Estimate of Average Relative Longitudinal Speed: $|\Delta\overline{V}|$

$|\Delta\overline{V}|$ is the average relative longitudinal speed between aircraft flying in the same direction. We estimate it from the TSD by taking the differences between the speeds of all the pairs of aircraft that constitute a lateral proximate pair in the same direction (see Figure 4). $|\Delta\overline{V}|$ is estimated as the mean absolute value of all the calculated differences, which turns out to be 35.13632. We use the conservative value 36. Here we note that the lateral proximate pairs are already determined while estimating the parameter E_y (same).

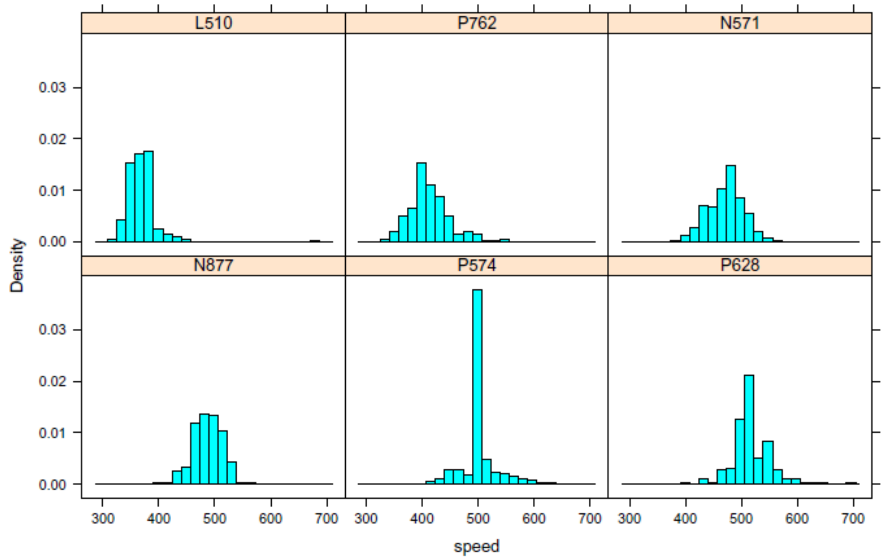

Figure 3: Distribution of estimated average speed by route.

2.9 Estimate of Average Relative Lateral Speed: $|\bar{\dot{y}}(S_y)|$

$|\bar{\dot{y}}(S_y)|$ is the average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by S_y NM at the same flight level, that have lost their lateral separation. The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study. So we take a conservative value 75 knots as per the EMA Handbook.

2.10 Estimate of Average Relative Vertical Speed: $|\bar{\dot{z}}|$

$|\bar{\dot{z}}|$ denotes the average modulus of the relative vertical speed between a pair of aircraft on the same flight level of adjacent tracks that has lost lateral separation. It is generally assumed that $|\bar{\dot{z}}|$ is independent of the size of the lateral separation between the aircraft and, for aircraft in level flight, it can also be considered that there is no dependency of $|\bar{\dot{z}}|$ with the vertical separation between the aircraft. As noted by various agencies data on $|\bar{\dot{z}}|$ are relatively scarce but typically taken as 1.5 knots which is considered to be conservative (see EMA Handbook).

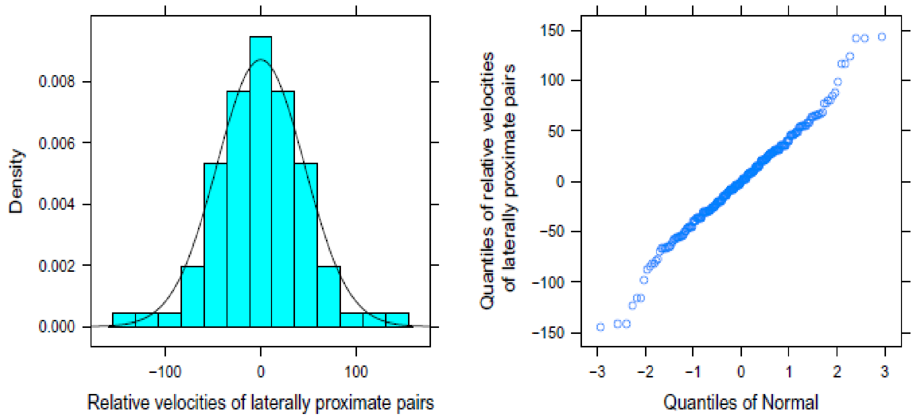

Figure 4: Distribution of relative velocities of laterally proximate pairs. The Normal distribution with sample standard deviation looks like a reasonable fit.

3. LONGITUDINAL COLLISION RISK ASSESSMENT

In order to compute the *level of safety* for longitudinal deviations of operations on the Bay of Bengal we use the *Longitudinal Collision Risk Model*. It models the longitudinal collision risk due to the loss of longitudinal separation between aircraft on adjacent flying on the same route at the same flight level. The model is as follows:

$$N_{ax} = P_y(0)\,P_z(0)\,\frac{2\lambda_x}{|\dot{x}|}\left(\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(0)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z}\right) \times \left[\sum_{k=m}^{M} Q(k)\,\mathbf{P}(K > k)\right]. \qquad (4)$$

We would like to note that the same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA.

The parameters in the equation (4) are defined as follows:

- N_{ax} :=Expected number of accidents (two for every collision) per flight hour due to the loss of longitudinal separation between co-altitude aircraft flying on the same track with planned minimum m NM longitudinal separation.
- m := Minimum longitudinal separation in NM.
- M := Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.
- λ_x := Average length of an aircraft flying on Bay of Bengal.
- λ_y := Average wingspan of an aircraft flying on Bay of Bengal.
- λ_z := Average height of an aircraft flying on Bay of Bengal.

- $P_y(0)$:= Probability that two aircraft assigned at the same route will be at same across-track position.
- $P_z(0)$:= Probability that two aircraft assigned to same flight level are at same geometric height.
- $|\bar{\dot{x}}|$:= Minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point.
- $|\bar{\dot{y}}(0)|$:= Relative across-track speed of same route aircraft pair.
- $|\bar{\dot{z}}|$:=Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.
- Q (k) := Proportion of aircraft pairs with initial longitudinal separation k.
- P (K > k) := Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Once again, a collision, and consequently two accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation 4 gathers the product of the probabilities of losing separation in each one of the three dimensions.

The equation is derived under similar assumption as done in the case of lateral collision risk assessment.

We should note here that the first part of the right-hand side of the equation (4) gives the probability of a collision given an event of overtake of a front aircraft by a behind aircraft when both are nominally flying at the same route at the same flight level, and the second part which is inside the square bracket is the probability of an overtake event.

3.1 Estimated Values of the Parameters and Estimated Longitudinal Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from our analysis.

Parameter	**Estimated Values**	**Source of the Estimate**
m	80 NM	Current minimum longitudinal separation.
M	160 NM	Conservative value corresponding to 20 minutes separation.
λ_x	0.0326051 NM	Estimated from TSD
λ_y	0.02983705 NM	Estimated from TSD
λ_z	0.009069301 NM	Estimated from TSD
$P_y(0)$	0.2	Conservative estimate (see Section 3.2).
$P_z(0)$	0.3617939	Estimated using a mixture model (see Section 2.5).
$\lvert\bar{\dot{x}}\rvert$	90 knots	Conservative estimate using speed and distance between way points (see Section 3.3)

$\lvert\bar{\dot{y}}(0)\rvert$	1 knot	RASMAG/9 safety assessment (see Section 3.4).
$\lvert\bar{\dot{z}}\rvert$	1.5	Conservative value as per EMA Handbook (see Section 2.10)
Q (k)	See Table 5	Obtained from TSD (see Section 3.5).
P (K > k)	See Table 5	Computed using normal model on speed(see Section 3.6).

Finally this leads to the following estimate for the longitudinal collision risk N_{ax}.

$$N_{ax} = 3.71804 \times 10^{-10}$$

k (mins)	k (NM)	Q(k)	P (K > k)
10	80	0.002235469	1.83061×10^{-6}
11	88	0.003353204	1.88145×10^{-7}
12	96	0.003725782	1.6016×10^{-8}
13	104	0.008196721	1.16613×10^{-9}
14	112	0.006706408	8.16394×10^{-11}
15	120	0.002608048	7.35331×10^{-12}
16	128	0.008941878	1.04974×10^{-12}
17	136	0.006333830	1.95268×10^{-13}
18	144	0.007451565	3.89188×10^{-14}
19	152	0.004843517	7.84075×10^{-15}
20	160	0.005961252	1.58302×10^{-15}

Table 3: Estimated values of Q (k) and P (K > k)

3.2 Estimating Probability of Lateral Overlap: Py (0)

P_y (0) is defined as the probability of lateral overlap of aircraft nominally flying at adjacent flight levels on same route. We can now use the same mixture model of Section 2.4 to compute this parameter by substituting $S_y = 0$ in the equation 2. This leads to an estimate of P_y (0) as 0.004527846.

However as noted earlier in the EUR/SAM report, this factor P_y (0,) has a significant effect on the risk estimate. Therefore, it should not be underestimated. P_y (0) will increase as the lateral navigational performance of typical aircraft improves, causing a corresponding increase in the collision risk estimate. As reported in the EUR/SAN report, the RGCSP was aware of this problem and attempted to account for improvements in navigation systems when defining the RVSM global system performance specification. Based on the performance of highly accurate area navigation systems observed in

European airspace, which demonstrated lateral path-keeping errors with a standard deviation of 0.3 NM, the RGCSP adopted a value of 0.059 as the value of $P_y(0)$ for the global system performance.
We further note that in the EMA Handbook the value has been taken conservatively as 0.2. We take this rather conservative value for our analysis as well.

3.3 Estimation of the Parameter $|\bar{\dot{x}}|$

$|\bar{\dot{x}}|$ is defined as the minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point. Thus if d is the distance between the two way points and v_0 is the speed of the front aircraft then $|\bar{\dot{x}}|$ can be computed by the equation

$$\frac{d-m}{v_0} = \frac{d}{v_0 + |\dot{x}|},$$

Leading to

$$|\bar{\dot{x}}| = \frac{mv_0}{d-m}.$$

We conservatively estimate it by taking v_0 as the minimum speed observed in TSD which is 315 NM per hour and the maximum distance between two waypoints on the routes which we study which is d = 338 NM. With m = 80 NM the final estimate turns out to be $|\bar{\dot{x}}|$ = 97.67442 knots. We use a conservative value of 90 knots.

3.4 Estimation of parameter $|\bar{\dot{y}}(0)|$

$|\bar{\dot{y}}(0)|$ is defined as the relative cross-track speed of same route aircraft pair. No data is available for estimation of this parameter so we take a conservative value of 1 knot as given in the EMA Handbook.

3.5 Estimation of the Parameter Q (k)

Q (k) is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the December 2010 TSD (Chennai FIR). Flights entering the FIR on different routes and assigned different flight levels were considered separately (see Figure 5), and the waiting times between successive arrivals were tabulated in minutes. We assume an average speed of 8 NM per minute, and compute the proportion Q(k) as

$$Q(k) = \frac{\text{number of flight pairs with inter-arrival distance 8k}}{\text{total number of flight pairs with at least 80 NM separation}}.$$

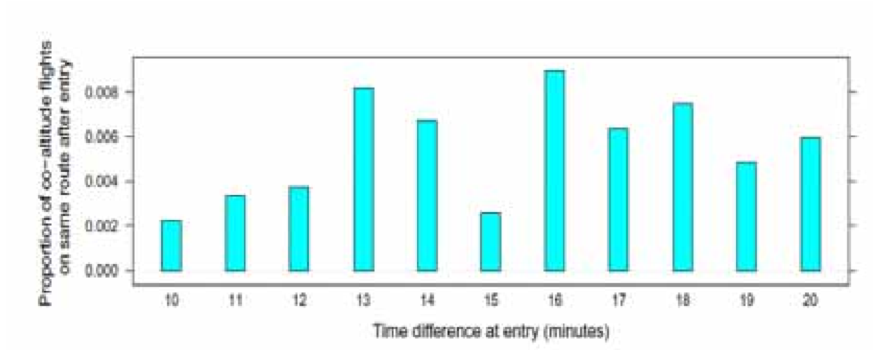

Figure 5: Values of Q(k) estimated from December 2010 Chennai FIR TSD.

For co-altitude flights on the same route (after entry / before exit), the proportion of flights that entered k minutes after the preceding flight is plotted for k = 10, 11, 12, . . . , 20 minutes. Note that the routes N877 and N571 have a (short) common segment, and flights entering or exiting at IGOGU do not follow the previous flight throughout their route. However, we have included these flights as well in order to be conservative. The final estimated value of Q (k) for k ranging between 10 and 20 minutes is given in the Table 3.

3.6 Estimation of the Parameter P (K > k)

To estimate P (K > k) we consider two aircraft flying on same route at same flight levels at the same direction. Let V and V' be their ground speeds of the front and behind aircraft respectively. We assume these speeds to be statistically independent but identically distributed. Let T_0 be the maximum duration of time before ATC intervenes. Then

$$\mathbf{P}\,(K > k) = \mathbf{P}\left(0 < \frac{k}{V' - V} < T_0\right) = \mathbf{P}\left(V' - V > \frac{k}{T_0}\right).$$

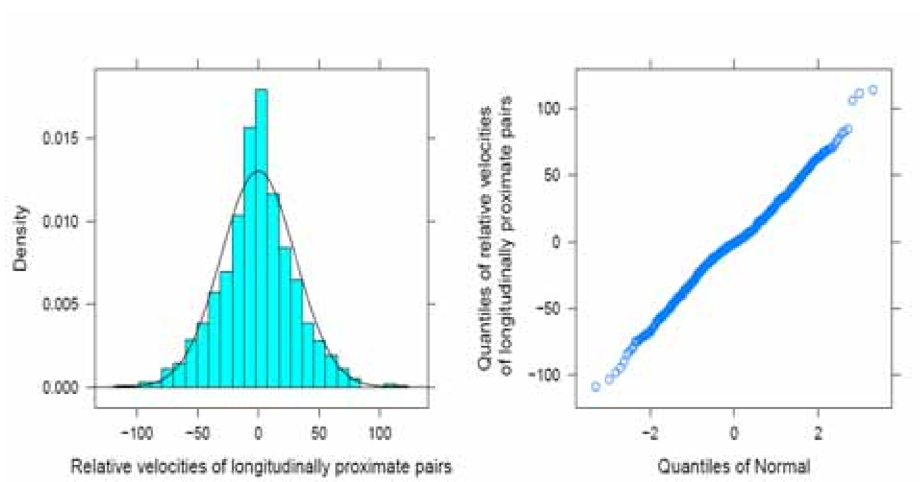

Figure 6: Distribution of relative velocities of longitudinally proximate pairs. The Normal distribution does not necessarily seem to be a reasonable fit.

We note here that the value of T_0 is conservatively taken to be 0.5 hours. Now we finally estimate these probabilities using the TSD. For that we consider the difference in velocity of two aircraft nominally flying on the same route at the same flight level. We conservatively consider velocity differences of all flight pairs which are separated by 2 hours time at entry. It is to be noted that we observed from the TSD data that two hours is more than the maximum time taken by any aircraft to travel between its entry and exit points. We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile- quantile plot (see Figure 6) it is not clear that these differences necessarily Normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v(v) = p\frac{\beta_v}{2}e^{-\beta_v|v|} + (1-p)\frac{1}{\sqrt{2\pi}\sigma_v}e^{-\frac{v^2}{2\sigma_v^2}},$$

which is a mixture of Double Exponential and Normal densities with mixing proportion p. We then estimate the parameters of this mixture model by their *maximum likelihood estimates (MLEs)*

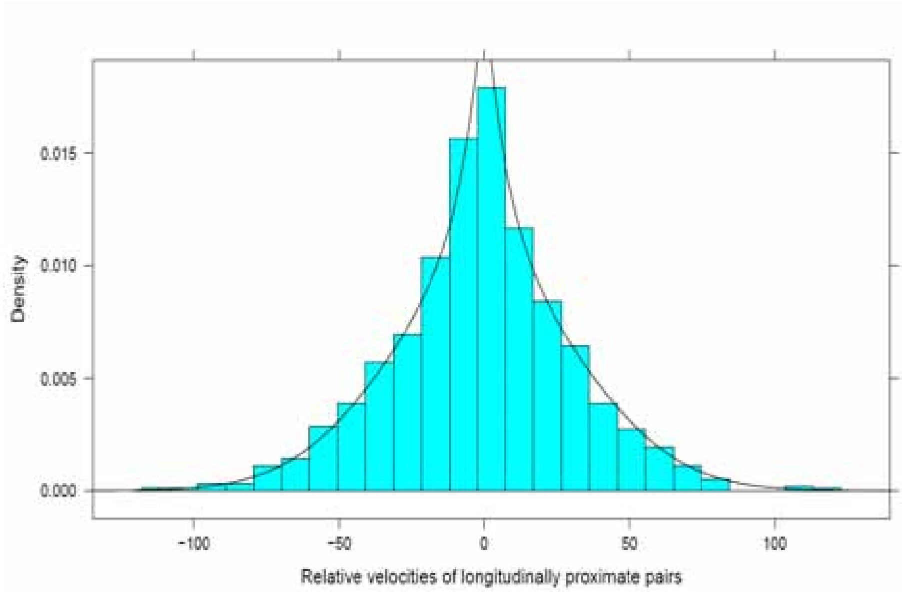

Figure 7: Distribution of relative velocities of laterally proximate pairs along with estimated mixture density (estimated using the EM algorithm).

Since this is a mixture model so we use the Expectation-Maximization (EM) algorithm to find the MLEs. The algorithm converged rapidly to give the following estimates:

$$\hat{p} = 0.2066880,\ \hat{\beta}_v = 0.1433969,\ \text{ and } \hat{\sigma}_v = 34.4067437\,.$$

It is well known in Statistics literature that even though the EM algorithm increases the value of the likelihood it may get trapped in a local maximum. To avoid this problem we tried several starting values and observed that the algorithm always converges to the same estimated values given above.

So it is statistically reasonable to accept the mixing density with these values of the parameters as a good estimate of the true density of the velocity differences. A graphical representation of the fit is given in Figure 7.

It is to be noted here that the final estimate of P (K > k) values will be more conservative if we increase the values of p and σ and decrease the value of β_v. Thus conservatively we take the estimated values as

$$\widehat{p} = 0.25, \widehat{\beta}_v = 0.1 \text{ and } \widehat{\sigma}_v = 35 .$$

With these we estimate the values of P (K > k) for k ranging between 10 and 20. These are presented in the Table 3.

References

1. EMA Handbook for Asia/Pacific Region Version-2 August 2010
2. Doc 9689, Manual on Airspace Planning Methodology for the Determination of separation Minima.
3. Annex 11 of the Convention on International Civil Aviation, Air Traffic Services.
4. Reports of Bay of Bengal Reduced Horizontal Separation Task Force Meeting.
5. Safety Assessment and Airspace Analysis of South China Sea
6. EUR/SAM: "Double Uni-directionality" Post-Implementation Risk Assessment
7. Aeronautical Information Publication (India).